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Moment–Curvature Relationship Prediction of Reinforced Concrete Beams by Adaptive Neuro-Fuzzy Inference System (ANFIS) Modelling

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Abstract Estimation of the moment–curvature (MC) relationship of reinforced concrete (RC) beam cross sections presents challenges due to the quite complex mixture of their composition. It is known that traditional MC calculation methods are time-consuming, require high computer power, and have a scarcity of data. The modern soft-computing models offer a reliable solution for precise estimation. This research proposes a new modified version of the adaptive neuro-fuzzy inference system (ANFIS) model that takes quite uncertainty source reduction in the estimation of the MC relationship that is valid for RC beam cross sections. The new approach provides an automatic deep-learning procedure to form a nonlinear MC relationship in the light of ANFIS software that accounts for uncertainty source reductions through fuzzy sets. For the application of this method, six input parameters are considered: cross-section width and height, concrete compressive and rebar yield strengths, total rebar area, and curvature. A normalization procedure is applied to transform each parameter into dimensionless amounts for comparison with existing models in the literature. The ANFIS model is prepared in such a way that training and test data percentages are taken as 75% and 25%, respectively. Finally, the comparison of the actual and proposed model output curves shows a ± 5% difference, which is a practically acceptable limit in practical applications. The best ANFIS computation results are achieved with three Gaussian fuzzy sets for each parameter and five fuzzy sets for curvature. It is recommended that this model can be improved by larger datasets also for different materials such as steel, timber, and prestressed concrete beam types to identify valid MC relationships of each material. Apart from the structural setup of methodological approaches, the innovative nature of this article is to eliminate the number of rule bases by taking into account expert opinions and obtaining the most meaningful outputs with the least percentage of errors.

Keywords Moment · Curvature · Reinforced concrete beam · Fuzzy · ANFIS

1 Introduction

One of the most critical issues in reinforced concrete (RC) construction engineering is identifying the mutual behaviour of moment–curvature (MC) in the form of functional relationships, especially for beam cross sections. Although there are different approaches in the literature, calculation takes a long time, even in high-speed computers with practically enough data. The MC curve has a nonlinear characteristic that provides stress capacity changes in terms of stress with strain. Takiguchi et al. [1] proposed initial studies for the analysis of RC cross sections subject to biaxial bending moments. Theoretical MC analysis of RC columns helps to indicate curvature ductility, rigidity, and strength, and it also provides a nonlinear stress–strain relationship by considering steel bar contributions. One can compute moment and curvature relation to flexure deformation increase in a column cross section for a set of axial loads that cause curvature increments under the strain compatibility and stress equilibrium according to nonlinear law. A reliable concrete stress–strain nonlinearity is significant for practically acceptable results confined

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within ± 5% relative error limits between the measured data and model MC curves. Mander et al. [2] stated the importance of acceptably accurate information about the confined concrete complete stress–strain curve for conducting reliable moment–curvature analysis to assess the ductility of columns with various transverse reinforcement arrangements.

Machine learning (ML) and artificial intelligence (AI) models, including classical and various artificial neural network (ANN) architectural structures, are the most frequently used alternatives with their single or few hidden layers. As Soroushian et al. [3] mentioned, different models are used to obtain the RC cross-section MC relationship. For example, several early studies were introduced by various researchers [4–9].

Mangalathu and Jeon [10] suggested the critical need for failure mode estimation for circular RC bridge columns by investigating the capabilities of ML methods. They mentioned the ML model efficiency analysis using second-order polynomial discriminant analysis, K-means, random forests, and ANN methods, which have superior performance among all the ML methods.

Shear strength estimation of RC beam–column joints is achieved by ML techniques [11]. On the other hand, Mangalathu et al. [12] used ML in terms of ANN for fragility assessment of bridges in California. Wang et al. [13] provided nuclear power plant vulnerability curves through ANN methodology. Unfortunately, many ML and AI versions of ANN are implemented mainly through ready software. Thus, some authors may not be aware of the scientific, philosophical, and logical background knowledge and information on the ML method. The agreement of the output data with the model estimation provides an acceptance rate. The following paragraph provides some studies on RC members related to ML methods in chronological order.

Haque [14] used ANN modelling for the preliminary design of RC beam-columns. The ANN back-propagation multilayered model was trained to estimate column cross sections based on input parameters, including concrete compressive strength, column types, steel ratio, axial load, and moment. Bagci [15] provided ANN methods to estimate the RC sections' MC relationships, which perform well for complex problem solutions with nonlinear material behaviours. Analytical data were used for ANN model training, and the estimations were compared with analytical results for a different set of input parameters that provided good agreement between the two approaches. Bilgehan [16] used ANFIS and ANN models to identify the relationship between concrete compressive strength and ultrasonic pulse velocity (UPV). Apart from the previous studies, concrete compressive strength estimation through the ANFIS model coupled with Gaussian membership functions (MFs) outperformed the ANN approach. ANFIS and multiple linear regression (MLR) models are used by Emiroglu et al. [17] for load measurement estimations of the peak pressure in glass fibre-RC pipes. Another fuzzy-genetic model to estimate the energy absorption capacity of beams is suggested by Altun et al. [18], which demonstrated practically acceptable performance in the applications. Their study highlighted the potential of hybrid fibre-RC and the effectiveness of the fuzzy-genetic model in estimating experimental results. Nasrollahzadeh and Basiri [19] proposed another approximate method concerning a fuzzy inference system (FIS) for the shear strength estimation of fibre-reinforced polymers reinforced concrete (FRP-RC) beams. Available design provisions for FRP-RC beams are often conservative or inadequate due to assumptions about shear-resisting mechanisms and the extension of steel-RC equations to FRP-RC beams. On the other hand, four ANN models to predict the residual flexural strength of corroded RC beams are developed by Imam et al. [20]. Ud Darain et al. [21] used ANFIS to estimate deflection and cracking behaviour in near surface-mounted (NSM) strengthened RC beams and demonstrated the potentiality of ANFIS as a reliably valid tool for the serviceability behaviour estimation of NSM-strengthened RC beams. Basarir et al. [22] explored different modelling techniques, including MLR and ANFIS, to estimate the ultimate pure bending moment of concrete-filled steel tubes (CFTs). Another fuzzy-based model is proposed by Naderpour and Alavi [23] for the estimation of the shear strength contribution of externally bonded fibre-reinforced polymer (FRP) sheets in RC beams. Their results showed the validity of the fuzzy-based model, which had good estimations that were practically acceptable for the shear contribution of FRP compared to existing guideline equations. Deflection behaviour estimation of plain, steel-RC, and bamboo-RC beams are investigated by Mishra et al. [24] through ANN models. Naderpour et al. [25] introduced the concept and application of a fuzzy logic-based model for the ultimate strength estimation of FRP-confined circular RC columns. An ANFIS model was developed using valid experimental data with seven input variables encompassing all parameters affecting the columns' compressive strength. Furthermore, Naderpour and Mirrashid [26] focussed on the structural vulnerability assessment during earthquakes with special concern for RC beam–column joints as a critical structure damage parameter. Instead of solely considering inter-story, the study employed two approaches, the group method of data handling (GMDH) and ANFIS, to determine the maximum shear capacity of RC joints. In their later study, Naderpour and Mirrashid [27] introduced an ANFIS estimative model with five inputs to determine the moment capacity of spiral-reinforced concrete columns. They trained and tested

the model through a database of 82 columns, and the ANFIS model demonstrated higher accuracy in the moment capacity estimations of spiral-reinforced concrete columns. They also investigated five methods based on computational intelligence (CI) to estimate the moment capacity in rectangular concrete columns, in which the equation-based neuro-fuzzy system provided high accuracy in the moment capacity estimation [28]. Park et al. [29] introduced a convolutional neural network-based (CNN) strain estimation technique for long-term structural health monitoring (SHM) in the absence or malfunction of strain sensors. Their method established dynamic structural and strain response relationships using CNNs. Bai et al. [30] focussed on the RC beams' deflection estimation problem using ensemble models based on ML and AI algorithms. They also provided six new hybrid model evaluations, including ANN-ANFIS, SVM-ANFIS, SVM-ANN, BA-ANFIS, BA-ANN, and BA-SVM, based on 120 experiments. Another set of AI techniques is used by Shariati et al. [31] to examine the tilted angle connectors (TAC) estimation as new shear connectors in steel-concrete composite systems. They have also used the ANFIS model to identify the most influential parameters on the shear strength of the connectors through different AI techniques, such as ANN, extreme learning machine (ELM), and ANFIS, for shear strength estimation. Cakiroglu et al. [32] studied AI alternatives, specifically data-driven ML algorithms, to estimate the axial capacity of FRP-RC columns with more reliable accuracy. Abushanab et al. [33] used ML techniques for the flexural capacity of RC beams affected by steel corrosion. Yaseen [34] investigated the shear strength mechanism of FRP-RC beams with transverse reinforcement using three ML models: M5-Tree, ELM, and random forest methodologies. The nine-input parameter model of M5-Tree yielded the best estimation performance with valuable insights into the impact of stirrups on FRP-RC beam shear strength. They also highlighted the potential of ML models for pre-design procedures in RC elements using FRP bars.

So far, there are not enough studies have been carried out on exploring the moment–curvature (MC) modelling capabilities of machine learning (ML) techniques for reinforced concrete (RC) beams by adaptive neuro-fuzzy inference system (ANFIS) methodology, which is the main subject of this paper. The primary purpose of this paper is to model MC curves using six input variables: section width, section height, concrete compression strength, rebar yield strength, rebar area, and curvature. The model's output MC function is the best practical confirmation with actual data of alternative conventional solutions within practically acceptable $\pm$ 5% error limits. In addition, one of the novel aspects of the proposed methodology is that it is more efficient in terms of calculation time and reliability of the results. The paper is structured as follows: Sect. 2 discusses the uncertainty sources in RC beams, Sect. 3 presents the reference MC analyses, Sect. 4 covers the ANFIS methodology, Sect. 5 provides the datasets and application details, and Sects. 6 and 7 include results, discussion, and conclusions, respectively.

2 Uncertainty Sources in RC Beams

Unreliable stability calculations in a civil engineering design can cause structural member damages, and thus, the safety criteria become very important for further investigations. Identification of uncertainty sources is necessary for refined methodological calculations, especially for material properties and design planning. There are software computation facilities based on standard regulation principles for practical applications. For better new research directions, they are not very helpful because one should know the philosophical and logical backgrounds of the model used before applying ready mathematical equations of software. Any researcher or responsible engineer should also control the uncertainty minimization possibility of the final product, especially in structures that must withstand additional external loads, for example, during an earthquake. In cases of overlooking the full importance of any design parameter, it may lead to destructive design consequences. After considering all the relevant scientific equations and guidance of standard engineering regulations, the engineer should feel responsible for taking into full account the design and calculation procedures with their possible uncertainty ingredients. This does not mean that the engineer is capable of avoiding all the uncertainties even though modern scientific and technological facilities are at his exposal. It is crucial, especially for a research engineer, to consider possible uncertainty sources in input and output design parameters. The priority lies in internal and external (e.g. seismic) loads, material properties, and structural design uncertainties. In structural design studies, RC strength is one of the most critical parameters because materials may have uncertainties affecting mechanical properties and durability [35].

There are different approaches for uncertainty evaluation in the literature, such as probabilistic, statistical, stochastic, and chaotic calculations, all depending on bivalent (crisp) logic. On the other hand, Zadeh [36–38] proposed his breakthrough innovative alternative as fuzzy logic, which considers numerical uncertainty sources and verbal vagueness, incompleteness, and ambiguities. Herein, fuzzy logic sets are considered for the uncertainty treatments, which can be supported by lexical (linguistic) expert views for better improvements in structural designs. In general, most frequently in the literature, statistical

uncertainty methodologies are used extensively in structural design studies [39, 40].

3 Reference Moment–Curvature (MC) Analyses

The reference MC analysis results are obtained by the OpenSees software [41]. A *ZeroLength* element was defined for each dataset, and the RC behaviour was represented through discretized fibre cross sections. For concrete and reinforcement elements (rebars), stress–strain relationships are represented by *UniaxialMaterial* objects, and the concrete stress–strain behaviour by the unconfined *Concrete02* material model [42]. The behaviour of the reinforcement was represented using the *Steel02* material model, which accounts for the isotropic strain hardening property of steel material. Regarding the relevant material and method, further details are explained by Filippou et al. [43].

MC analyses are continued until the maximum curvature value is reached. The maximum curvature is assumed to be six times the cross section's estimated yield curvature, κ_y (1), expressed by the following equation [43]:

$$\kappa_y = 0.7\varepsilon_y d' \tag{1}$$

where ε_y is the yield strain and d' is the calculation depth of the section.

4 ANFIS Methodology for RC Model

In general fuzzy logical (FL) model applications, there are two fuzzy inference systems (FIS), such as Mamdani [44, 45] and Sugeno [46]. Both FISs have input fuzzy rules and output layers. For example, Mamdani FIS has input and output layers in terms of complete fuzzy sets, and thus, it is regarded as a holistic fuzzy methodology. On the other hand, Sugeno FIS has input layers in fuzzy set forms, but the output is in the form of either a constant or linear equation in terms of input variables, and thus, it is referred to as a half-fuzzy methodology. Artificial-neuro-fuzzy inference system (ANFIS) is like Sugeno's [46] approach, but instead of one layer between input and output layers, there are four additional layers, as explained below.

As mentioned in the Introduction section and Sect. 2, RC cross-section MC computation is quite complex because it requires a very long digital computer execution time, and the used mathematical formulations are without logical foundation explanations. Each input and output variable has linguistic and numerical uncertainty that well-known mathematical equation sets cannot quantify. In order to deal with such sophisticated and complex uncertainties, it is necessary to deal with them by utilizing an effective uncertainty methodology, which is adopted in this research as the ANFIS method. The general architectural structure of the ANFIS model is shown in Fig. 1, distinctively from the existing literature through six layers.

Herein, each input variable uncertainty is represented by three to five fuzzy sets (FSs) in the form of membership degrees (MDs) in the proposed ANFIS methodology. The general architectural structure of ANFIS methodology is given in Fig. 2, again in a new form with five successive stages.

Figure 2 represents only two input variables for a simple conceptual explanation. If the reader understands the function of each layer, then it is not difficult to conceptualize six input variables. Interested readers can consult the book by Ross [47] for detailed information. A brief explanation of the layers is presented below.

(1) **Input layer:** As already stated in the Introduction section, six variables' data (section width, section height, concrete compression strength, rebar yield strength, rebar area, and curvature) are entered in sequence numerically,

(2) **Fuzzy set layer:** The first five variables are fuzzified by three fuzzy sets: "LOW", "MEDIUM", and "HIGH". The curvature input is fuzzified by five fuzzy sets linguistically as "VERY LOW", "LOW", "MEDIUM", "HIGH", and "VERY HIGH". The input variable numerical values trigger at least two of these fuzzy sets, each with a membership degree (MD),

(3) **Membership degree combination layer:** MDs from each variable are combined with AND junction so that their multiplication represents the joint behaviour of input variables in the fuzzy domain. In Figs. 1 and 2, the mathematical form of MD multiplications are given as weights, w_1 and w_2,

(4) **Weight percentage layer:** This stage determines the influence of each joint variable impact percentage in the ANFIS methodology calculation for each variable. The summation of all percentages is equal to 1. The third layer in Fig. 2 shows the percentage calculation in the rectangular boxes,

(5) **Individual variable estimation layer:** At this stage, input variable values are multiplied by corresponding percentages, and hence, percentage contribution amounts for each input variable are calculated, denoted by f_1 and f_2, in the following slimmer rectangular boxes.

(6) **Output layer:** The input variable percentages in the previous step are added to find the estimation of the final output value.

The next step is the calculation of the relative error between the model output and the numerically available

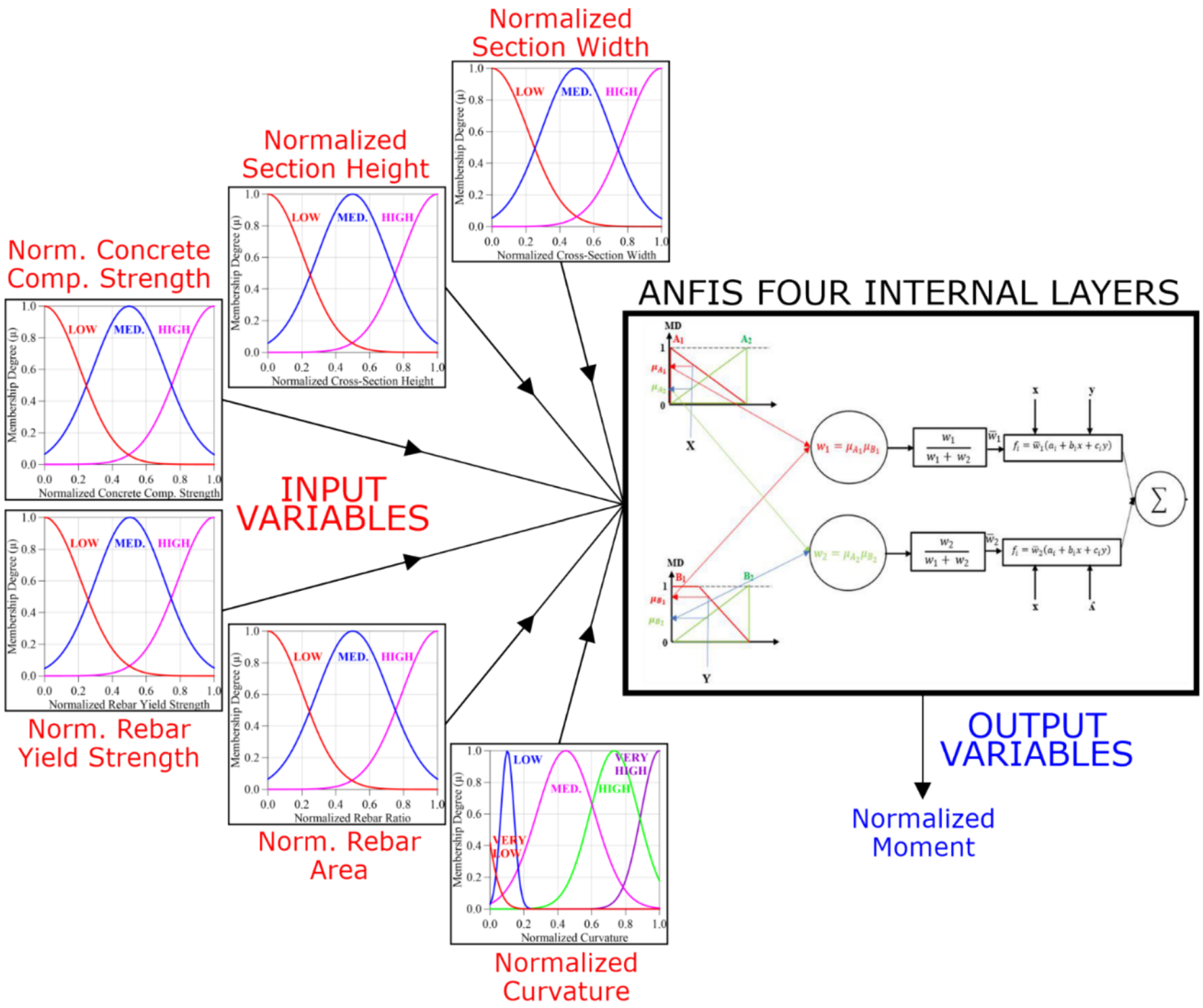


Fig. 1 ANFIS Architecture

data. If the error is not within the desired limits, the ANFIS procedure reiterates the calculations by changing the position of the input variable fuzzy sets until the ± 5% relative error amount is reached.

4.1 Membership Functions (MFs)

Different types of fuzzy sets (FSs) are available in the MATLAB [48] software, and herein for the ANFIS training methodology, Gaussian MFs are adapted. The mathematical form of the Gaussian MF is in the form of a bell shape with a central location parameter, c, and variation, σ, around this centre. The general form of a Gaussian MF function, f(x) (2), is given below [37].

$$f(x) = e^{-(x-c)^2/2\sigma^2} \tag{2}$$

Here, x is representative of input and output variables. Although three MFs are used for the first five input variables, five MFs are considered for the curvature input variable. MF parameters are given collectively for each input variable in Table 1.

Before entering each input variable data into the proposed ANFIS model, each is normalized so as to have a dimensionless variability confined within a range from 0 to 1. The general form of the normalization procedure (3) is given in the following expression [47].

$$\text{Normalized}\,(x) = x' = \frac{x - \min(x)}{\max(x) - \min(x)} \tag{3}$$

The fuzzy sets are shown in Fig. 3, with three to five descriptions. "LOW", "MEDIUM", and "HIGH" fuzzy sets are used in the first five input variables, while the last input variable, curvature, five fuzzy sets are used as

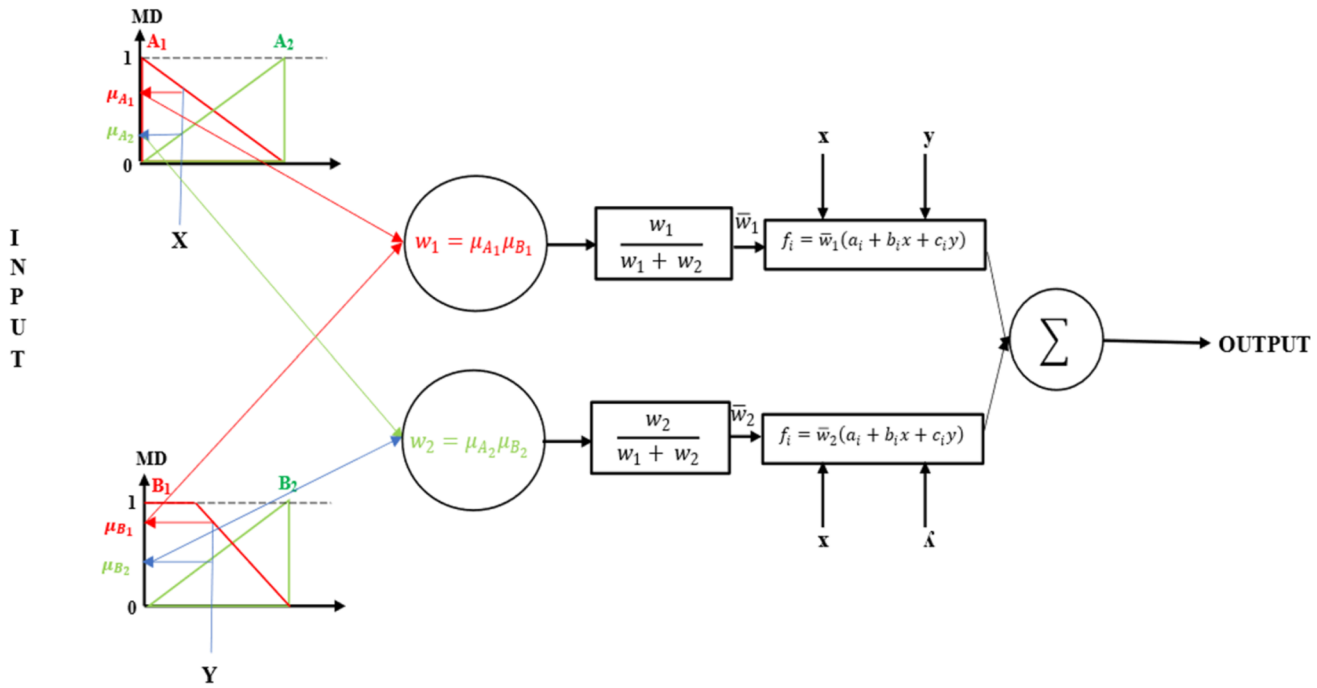


Fig. 2 ANFIS Six-layer model

Table 1 Gaussian MF parameters

Input variable	Membership function	c	σ
	LOW	0.212	− 5.447e-05
Cross-section width (b)	MEDIUM	0.205	0.497
	HIGH	0.212	0.999
	LOW	0.209	− 0.001
Cross-section height (h)	MEDIUM	0.209	0.498
	HIGH	0.212	1.000
	LOW	0.210	− 0.002
Concrete comp. strength (f_c)	MEDIUM	0.213	0.498
	HIGH	0.213	1.000
	LOW	0.212	1.838e-07
Rebar yield strength (f_y)	MEDIUM	0.203	0.503
	HIGH	0.212	1.000
	LOW	0.207	− 0.003
Rebar area (A_s)	MEDIUM	0.214	0.500
	HIGH	0.211	1.001
	VERY LOW	0.084	− 0.111
	LOW	0.039	0.103
Curvature (κ)	MEDIUM	0.170	0.446
	HIGH	0.143	0.733
	VERY HIGH	0.108	0.999

"VERY LOW", "LOW", "MEDIUM", "HIGH", and "VERY HIGH".

4.2 The Fuzzy Logic Rule Base

Logical rules are in the form of propositions in open literature with their two parts as precedence and consequent

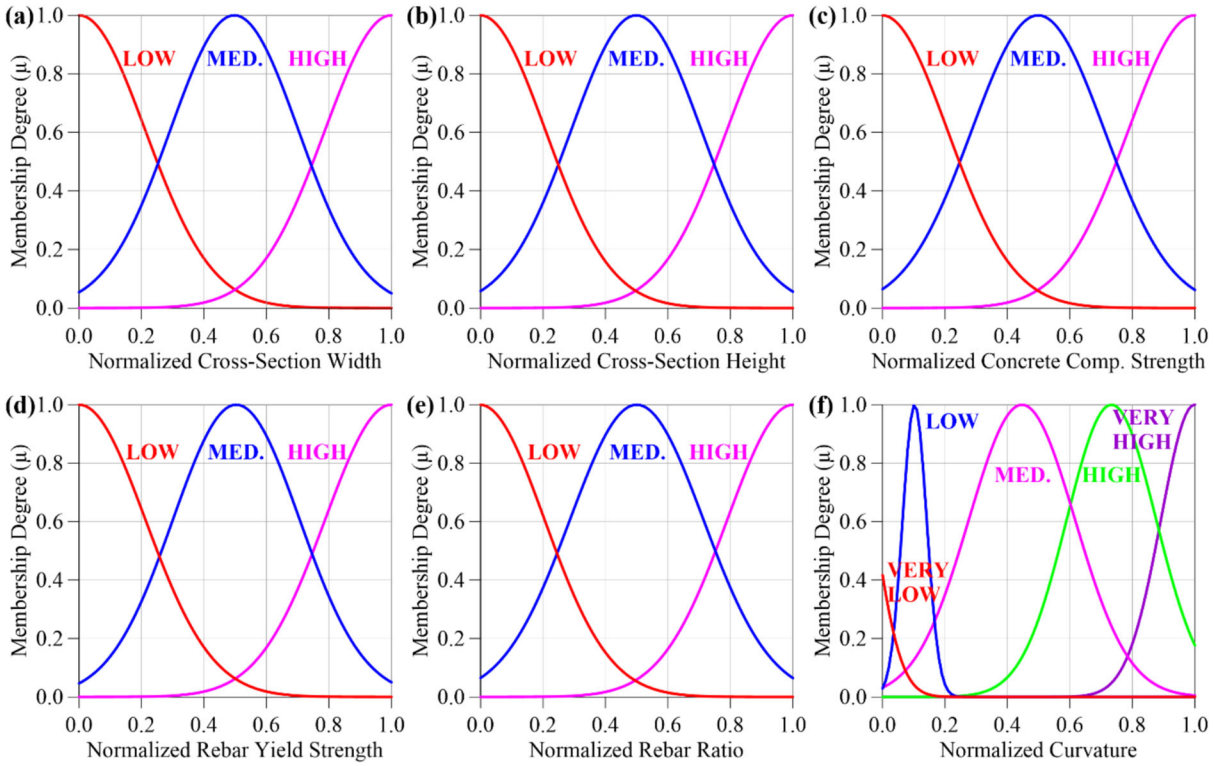


Fig. 3 MFs for normalized values of **a** cross-section width, **b** cross-section height, **c** concrete compression strength, **d** rebar yield strength, **e** rebar area, and **f** curvature

[37, 47]. In common terms, each fuzzy rule represents partially cause–result relationships depending on the combination of input variable fuzzy set combinations. In the antecedent (causatives) part, all input variables are connected by AND junction, and the number of rules depends on the multiplication of input variables' MFs numbers. Since there are six variables, the first each with three and the last one with five MFs, the number of automatic fuzzy rules is equal to $3 \times 3 \times 3 \times 3 \times 3 \times 5 = 1215$. Some of these rules do not have any valid scientific expert information, and thus must be eliminated from the system. However, this is automatically done by the proposed ANFIS program. In this paper, prior to the elimination of the ANFIS fuzzy rules, the number of fuzzy rules is reduced to an expertly acceptable level depending on the authors' and other experts' views. This point is very important for the application of the ANFIS program. Otherwise, it may lead to over or under-estimation without expert views. In order to provide some samples to the readers, the following few fuzzy rules are given, and the rest can be perceived and understood.

IF b is "Low" AND h is "Medium" AND f_c is "Low" AND f_y is "Low" AND A_s is "High" AND κ is "Medium" THEN M is O_1.

OR

IF b is "High" AND h is "Low" AND f_c is "Medium" AND f_y is "Low" AND A_s is "Low" AND κ is "High" THEN M is O_2.

OR

IF b is "Medium" AND h is "High" AND f_c is "Low" AND f_y is "High" AND A_s is "Low" AND κ is "Very High" THEN M is O_3.

OR

IF b is "High" AND h is "Low" AND f_c is "Low" AND f_y is "High" AND A_s is "Medium" AND κ is "Very Low" THEN M is O_4.

OR

*

*

*

*

IF b is "High" AND h is "High" AND f_s is "Low" AND f_y is "Low" AND A_s is "Medium" AND κ is "Very Low" THEN M is O_{1215}.

M stands for the output in each fuzzy rule output, meaning ANFIS Normalized Moment. Subsequent fuzzy rules are connected with the OR junction because either one or both can be valid for modelling purposes.

The output of each fuzzy rule is shown by O_i (i = 1, 2,..., 1215). In this research, each one (4) is considered as a partial linear line, similar to the Sugeno [46] Method.

$$O_i = a_i + b_i M \tag{4}$$

where a_i and b_i are the intercept and slope parameters for each part of the output numerical data variation domain for the precedent parts of the fuzzy rules, and M is the numerical value that is available as output data.

The execution of the fuzzy rule base is achieved by the ANFIS program, which automatically finds valid rules for the collective entrance of six variable numerical values and accordingly attaches a partially linear portion as the output value. This automation program of ANFIS continues iteration until the overall relative error average lies within the limits of ± 5% error.

5 Datasets and Application

The dataset consists of rectangular, singly-reinforced RC sections with different geometries and material properties. Characteristic section properties are represented by five different parameters that are section width (b), section height (h), concrete compression strength (f_c), yield strength of rebars (f_y), and rebar area (A_s). Table 2 provides the numerical values corresponding to these five parameters. Rebar area values are calculated according to the rebar ratio, R, which corresponds to the division of total rebar area to total cross-sectional area. Rebar ratios for cases within this study are adapted as 0.003 and 0.005 (R3 and R5). Reference Moment (M_{ref}) and curvature (κ) values are found by the valid method presented in Sect. 3. The variables related to curvature input and moment outputs are given in Table 2.

The dataset is diversified by utilizing these different numerical values, resulting in a total of 95 different reinforced concrete sections. Among them, 70 were used for training, while the remaining 25 were used for testing. The testing dataset was randomly selected among 95 sections. Table 3 shows the properties of training dataset sections, while Table 4 is for the test data. Each MC graph is formed with 15 data points; hence, the training dataset is formed with 1050 rows that contain six inputs and one output (reference moment), and the test dataset is formed with 375 data rows.

6 Results and Discussions

In light of what has been explained in the previous three sections, the application of the ANFIS methodology is run with six input and one output variable. The final outputs from the model and those based on the reference test data are presented in Figs. 4 and 5 for 25 test cases in the form of MC curves. Generally, the agreement between the model output and data MCs is acceptable with less than + 5% mean relative error (MRE) percentages. Due to data heterogeneity or outlier data, few cases do not conform to the reference MC curve. The moment–curvature relationships obtained for the sections in the test dataset using ANFIS are presented in comparison with the reference MC relationships.

The sum of the mean relative errors (MRE) between the ANFIS and reference test analyses is calculated according to the mean relative error (MRE) percentage definition given in the following expression (5) to determine a single parameter encompassing the entire MC relationship [49].

$$MRE(\%) = \frac{\sum[|M_{\text{anfis}} - M_{\text{ref}}| / \max(M_{\text{anfis}}, M_{\text{ref}})]}{N} \times 100 \tag{5}$$

where M_{anfis} and M_{ref} are the ANFIS and reference moment values, respectively. N is the total number of data points for a single analysis, which is 15 for each section in this study. Table 5 and Fig. 6 show the error values for each test

Table 2 Dataset parameters

Property	Symbol (unit)	Variables
Section width	b (mm)	150, 250, 300
Section height	h (mm)	400, 500, 600
Concrete compression strength	f_c (MPa)	10, 15, 20, 25, 30
Rebar yield strength	f_y (MPa)	220, 420
Rebar area	A_s (mm^2)	180 ~ 900
Curvature	κ (rad/m)	0 ~ 5.38E-05
Reference moment	M_{ref} (kNm)	0 ~ 211
ANFIS moment	M_{anfis} (kNm)	0 ~ 242 (see Sect. 6)

Table 3 Training dataset

No –	Abbreviation –	b mm	h mm	f_c MPa	f_y MPa	A_s mm^2	No –	Abbreviation –	b mm	h mm	f_c MPa	f_y MPa	A_s mm^2
1	150–400-C10-S220-R3	150	400	10	220	180	36	250–500-C25-S220-R3	250	500	25	220	375
2	150–400-C10-S220-R5	150	400	10	220	300	37	250–500-C25-S220-R5	250	500	25	220	625
3	150–400-C10-S420-R5	150	400	10	420	300	38	250–500-C25-S420-R5	250	500	25	420	625
4	150–400-C15-S220-R3	150	400	15	220	180	39	250–500-C30-S220-R3	250	500	30	220	375
5	150–400-C15-S420-R5	150	400	15	420	300	40	250–500-C30-S220-R5	250	500	30	220	625
6	150–400-C20-S220-R3	150	400	20	220	180	41	250–500-C30-S420-R5	250	500	30	420	625
7	150–400-C20-S420-R3	150	400	20	420	180	42	300–500-C10-S220-R3	300	500	10	220	450
8	150–400-C20-S420-R5	150	400	20	420	300	43	300–500-C10-S220-R5	300	500	10	220	750
9	150–400-C25-S220-R3	150	400	25	220	180	44	300–500-C10-S420-R3	300	500	10	420	450
10	150–400-C25-S220-R5	150	400	25	220	300	45	300–500-C10-S420-R5	300	500	10	420	750
11	150–400-C25-S420-R3	150	400	25	420	180	46	300–500-C15-S220-R3	300	500	15	220	450
12	150–400-C30-S220-R3	150	400	30	220	180	47	300–500-C15-S420-R3	300	500	15	420	450
13	150–400-C30-S220-R5	150	400	30	220	300	48	300–500-C15-S420-R5	300	500	15	420	750
14	150–400-C30-S420-R3	150	400	30	420	180	49	300–500-C20-S220-R3	300	500	20	220	450
15	150–500-C10-S220-R3	150	500	10	220	225	50	300–500-C20-S220-R5	300	500	20	220	750
16	150–500-C10-S220-R5	150	500	10	220	375	51	300–500-C20-S420-R3	300	500	20	420	450
17	150–500-C15-S220-R3	150	500	15	220	225	52	300–500-C25-S220-R3	300	500	25	220	450
18	150–500-C15-S220-R5	150	500	15	220	375	53	300–500-C25-S220-R5	300	500	25	220	750
19	150–500-C15-S420-R3	150	500	15	420	225	54	300–500-C30-S220-R3	300	500	30	220	450
20	150–500-C15-S420-R5	150	500	15	420	375	55	300–500-C30-S220-R5	300	500	30	220	750
21	150–500-C20-S220-R5	150	500	20	220	375	56	300–500-C30-S420-R5	300	500	30	420	750
22	150–500-C20-S420-R3	150	500	20	420	225	57	300–600-C10-S220-R3	300	600	10	220	540
23	150–500-C20-S420-R5	150	500	20	420	375	58	300–600-C10-S420-R3	300	600	10	420	540
24	150–500-C25-S420-R3	150	500	25	420	225	59	300–600-C10-S420-R5	300	600	10	420	900
25	150–500-C30-S220-R3	150	500	30	220	225	60	300–600-C15-S220-R3	300	600	15	220	540
26	150–500-C30-S220-R5	150	500	30	220	375	61	300–600-C15-S220-R5	300	600	15	220	900
27	150–500-C30-S420-R5	150	500	30	420	375	62	300–600-C15-S420-R5	300	600	15	420	900
28	250–500-C10-S220-R3	250	500	10	220	375	63	300–600-C20-S220-R3	300	600	20	220	540
29	250–500-C10-S220-R5	250	500	10	220	625	64	300–600-C20-S220-R5	300	600	20	220	900
30	250–500-C10-S420-R5	250	500	10	420	625	65	300–600-C20-S420-R3	300	600	20	420	540
31	250–500-C15-S220-R3	250	500	15	220	375	66	300–600-C25-S220-R3	300	600	25	220	540
32	250–500-C15-S420-R3	250	500	15	420	375	67	300–600-C25-S220-R5	300	600	25	220	900
33	250–500-C20-S220-R3	250	500	20	220	375	68	300–600-C25-S420-R3	300	600	25	420	540
34	250–500-C20-S220-R5	250	500	20	220	625	69	300–600-C30-S220-R3	300	600	30	220	540
35	250–500-C20-S420-R3	250	500	20	420	375	70	300–600-C30-S420-R3	300	600	30	420	540

section. The average of MREs between the proposed model MC curve and reference test dataset is found to be 3.55%.

The data scatter chart of M_{ref} vs M_{anfis} values of all data points is presented in Fig. 7 to show the coherence between reference and ANFIS moment values. In Fig. 7, there are 450 scatter points from all test case runs around the perfect line of 1:1 (45°) slope. The scatter points remain within the ± 5% and ± 10% errors, as shown between parallel lines to the perfect line. The majority of the scatter points show that the proposed ANFIS model is bound to make under-estimations, but the percentages of them are about 100 × (12/450) = 2.67% and 100 × (2/450) = 0.44% at −

Table 4 Test dataset

No –	Abbreviation –	b mm	h mm	f_c MPa	f_y MPa	A_s mm^2	No –	Abbreviation –	b mm	h mm	f_c MPa	f_y MPa	A_s mm^2
1	150–400-C15-S220-R5	150	400	15	220	300	13	250–500-C15-S420-R5	250	500	15	420	625
2	150–400-C15-S420-R3	150	400	15	420	180	14	250–500-C20-S420-R5	250	500	20	420	625
3	150–400-C20-S220-R5	150	400	20	220	300	15	250–500-C25-S420-R3	250	500	25	420	375
4	150–500-C10-S420-R3	150	500	10	420	225	16	250–500-C30-S420-R3	250	500	30	420	375
5	150–500-C10-S420-R5	150	500	10	420	375	17	300–500-C15-S220-R5	300	500	15	220	750
6	150–500-C20-S220-R3	150	500	20	220	225	18	300–500-C20-S420-R5	300	500	20	420	750
7	150–500-C25-S220-R3	150	500	25	220	225	19	300–500-C25-S420-R3	300	500	25	420	450
8	150–500-C25-S220-R5	150	500	25	220	375	20	300–500-C25-S420-R5	300	500	25	420	750
9	150–500-C25-S420-R5	150	500	25	420	375	21	300–500-C30-S420-R3	300	500	30	420	450
10	150–500-C30-S420-R3	150	500	30	420	225	22	300–600-C10-S220-R5	300	600	10	220	900
11	250–500-C10-S420-R3	250	500	10	420	375	23	300–600-C15-S420-R3	300	600	15	420	540
12	250–500-C15-S220-R5	250	500	15	220	625	24	300–600-C20-S420-R5	300	600	20	420	900
							25	300–600-C30-S220-R5	300	600	30	220	900

5 and − 10% levels, which are practically acceptable error limits, respectively.

Figure 8 shows the fuzzy rule partial surfaces in three dimensions between two independent input variables with the output normalized moment variations. Interpretation of each graph in Fig. 8 may take many pages; thus, only three selective graphs are explained. For example, the first graph on the left-hand side in the first row indicates the normalized variables' triple relationship between two input variables: section width and height and output normalized moment. It shows that "LOW" and "HIGH" section height does not have a significant effect on the moment, but "MEDIUM" values are comparatively more effective. The impact of section width on the moment continuously increases, starting from "LOW" values and then "MEDIUM" values. The most effective part lies in the "HIGH" width with "MEDIUM" height. Another graph is from the third row, middle part, which shows the change or normalized moment value changes with respect to section height and rebar area. The most effective part of the moment appears in "MEDIUM" fuzzy set height and rebar area combination values. Finally, the relationships between the normalized moment and curvature and rebar area are shown in the last row and the last graph on the right-hand side. One can conclude that normalized moment change is very "HIGH" at the "MEDIUM" section height with almost all classes of curvature, namely, "LOW", "MEDIUM", and "HIGH". Other graphs can be interpreted in the same way. Thus, detailed information about the effects of the combination of six input and the moment variables can be obtained.

7 Conclusions

The use of machine learning (ML) and artificial intelligence (AI) approaches, especially in terms of artificial neural networks (ANNs), started from 1990 onwards at an exponential rate and reached today's level in applications of many engineering problem solutions. These have recently been collected under the umbrella of deep-learning (DL) methodology. Unfortunately, many published papers are unaware of shallow learning principles, which are the etymological and epistemological science philosophical and logical rule bases in problem-solving, because overwhelmingly published papers are based on crisp (two-value, bivalent) logic with scarce applications by the employment of fuzzy logic inference (FIS) modelling principles. This paper proposes a detailed explanation and application of ANFIS for fast computation of moment–

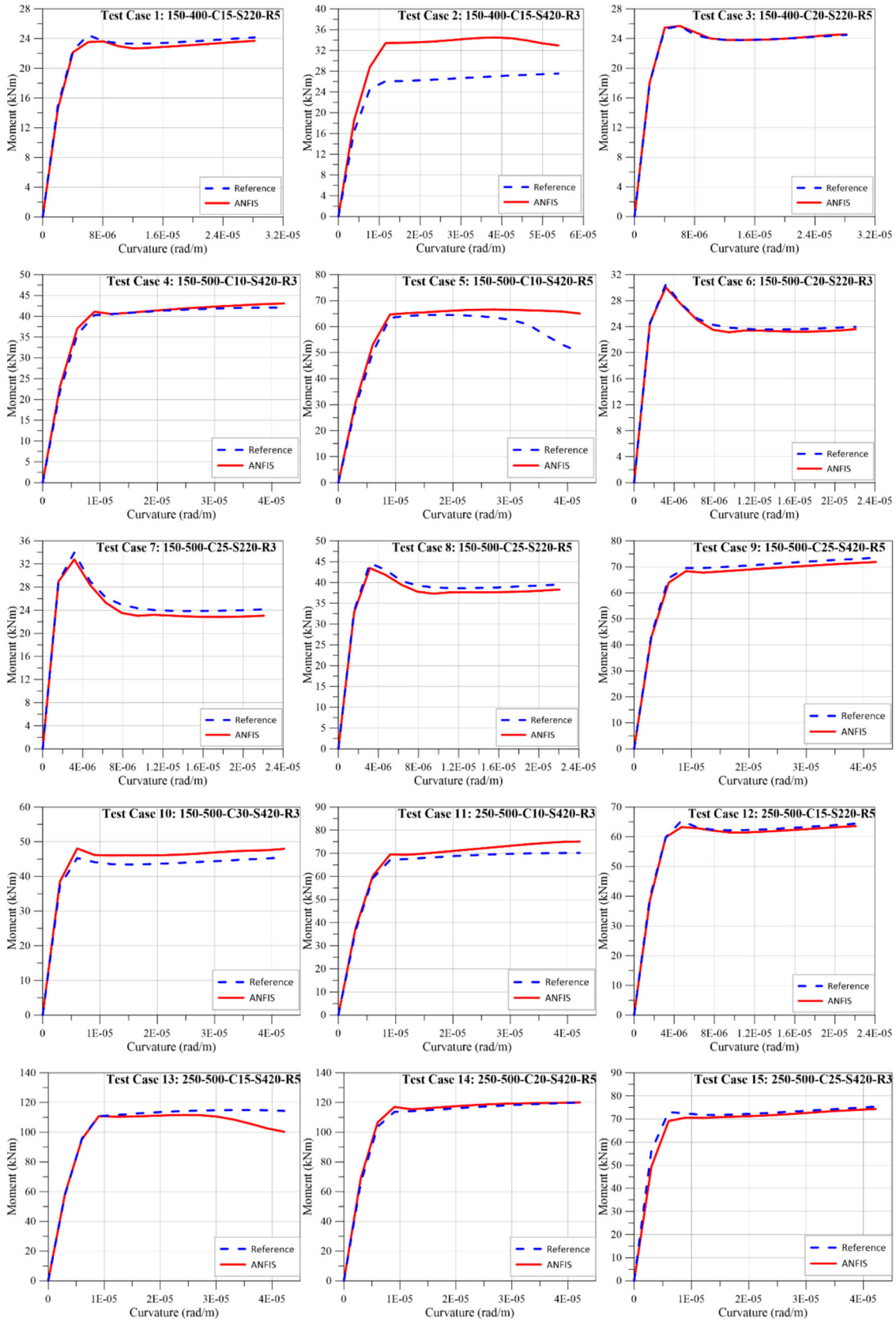


Fig. 4 Comparison of moment–curvature relationships for test dataset; test cases 1 ∼ 15

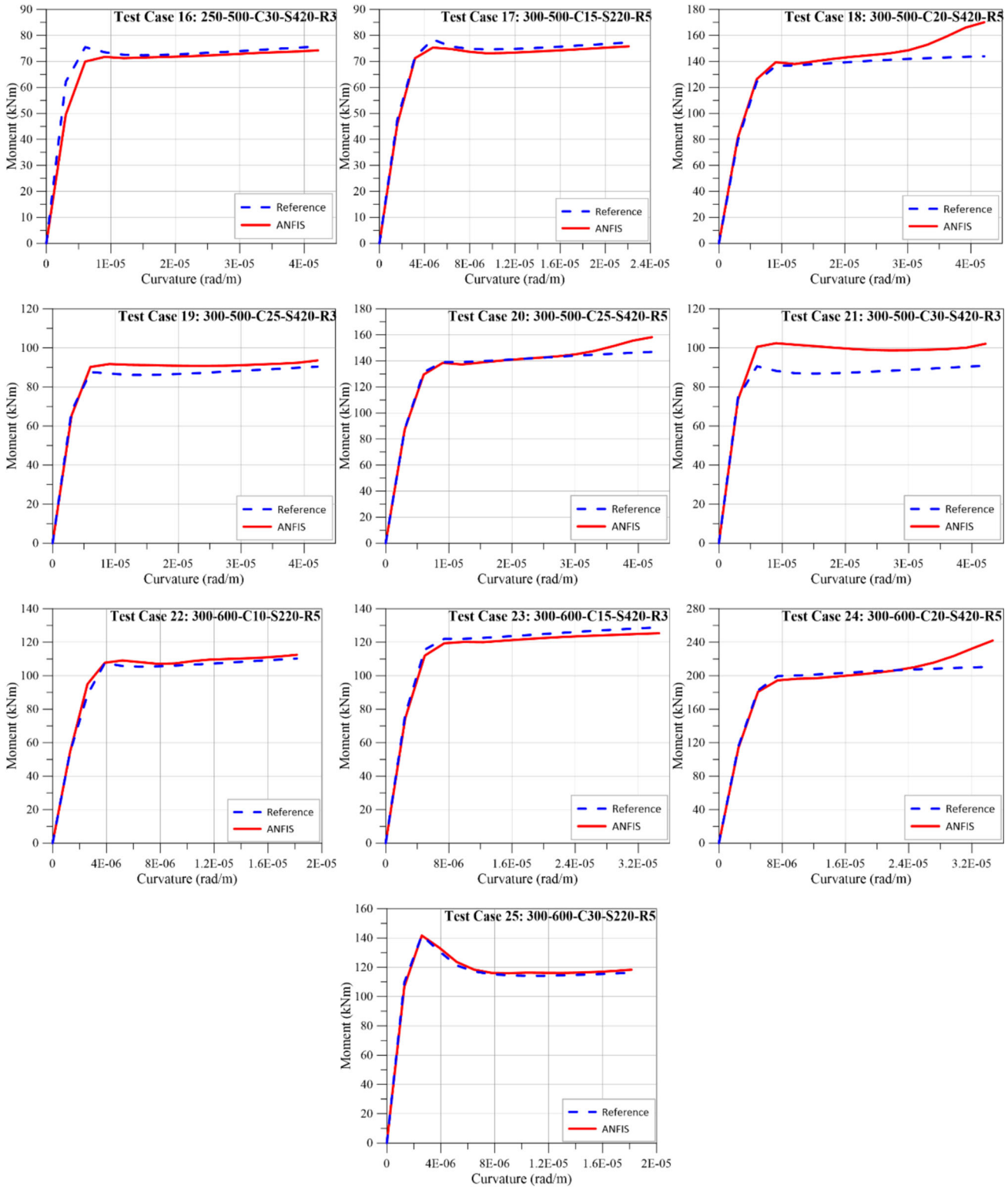


Fig. 5 Comparison of moment–curvature relationships for test dataset; test cases 16 ∼ 25

Table 5 Mean relative errors between ANFIS and reference test dataset

No	Abbreviation	MRE (%)	No	Abbreviation	MRE (%)
1	150–400-C15-S220-R5	1.93	14	250–500-C20-S420-R5	1.27
2	150–400-C15-S420-R3	18.40	15	250–500-C25-S420-R3	2.32
3	150–400-C20-S220-R5	0.27	16	250–500-C30-S420-R3	3.14
4	150–500-C10-S420-R3	1.44	17	300–500-C15-S220-R5	1.88
5	150–500-C10-S420-R5	6.62	18	300–500-C20-S420-R5	4.68
6	150–500-C20-S220-R3	1.40	19	300–500-C25-S420-R3	3.64
7	150–500-C25-S220-R3	3.57	20	300–500-C25-S420-R5	1.64
8	150–500-C25-S220-R5	2.66	21	300–500-C30-S420-R3	10.07
9	150–500-C25-S420-R5	2.08	22	300–600-C10-S220-R5	1.89
10	150–500-C30-S420-R3	4.81	23	300–600-C15-S420-R3	2.13
11	250–500-C10-S420-R3	3.78	24	300–600-C20-S420-R5	3.19
12	250–500-C15-S220-R5	1.16	25	300–600-C30-S220-R5	1.35
13	250–500-C15-S420-R5	3.56		Average:	3.55

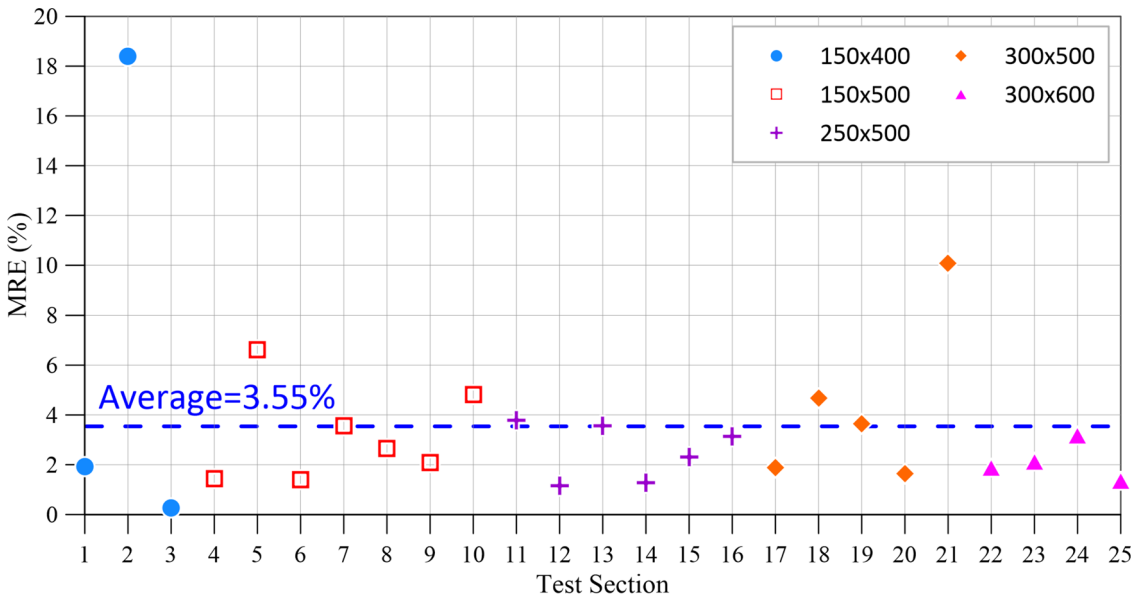


Fig. 6 Mean relative errors of the test dataset

curvature (MC) functional relationship with six input parameters (section width, section height, concrete compression strength, rebar yield strength, rebar area, and curvature). It is well known in the literature that the MC solution is very complex, and achieving a practically acceptable ± 5% relative error is tedious. This is the main reason why fuzzy logic rule-based treatment is aimed at MC output relationship search in this paper. The allegiance of the proposed methodology is the consideration of uncertainty components in the input and output variables by fuzzy set attachment, fast computation time, and detailed exposition of the proposed ANFIS method. The innovative and partially innovative points of the proposed ANFIS model to estimate the MC relationship include the following points.

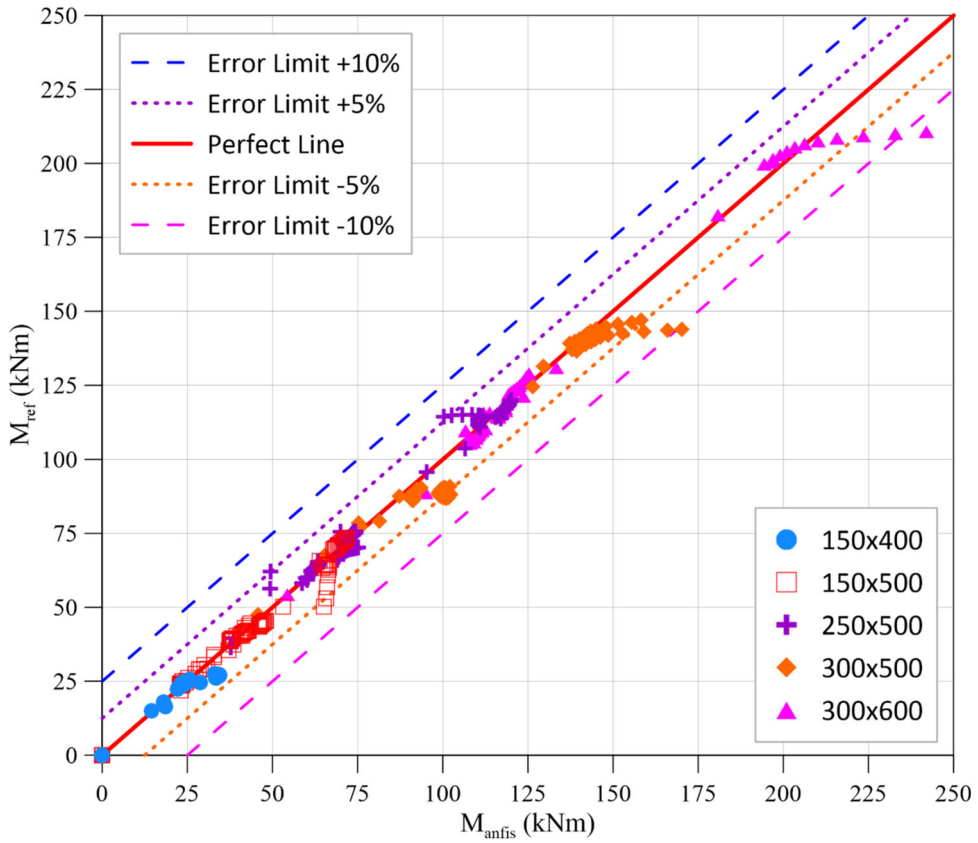


Fig. 7 Scattered data charts of M_{ref} versus M_{anfis} for different section geometries

- The computational time is decreased, which is comparatively shorter than existing methodologies for MC calculations.
- The uncertainty in the input parameters is addressed by using fuzzy sets and validating the fuzzy rule base between variables.
- Less than $\pm$ 5% relative error possibility is achieved.

It is recommended that the ANFIS approach be used successfully in future works in light of the text explained in this paper. The scope of cross-section analysis can be expanded by increasing the input parameters considering the methodologically flexible nature of the ANFIS approach with further finer explanations. In the future, this method can also be used to analyse the MC behaviour of steel, timber, or prestressed beams and RC sections with confined core concrete or subjected to axial force effects.

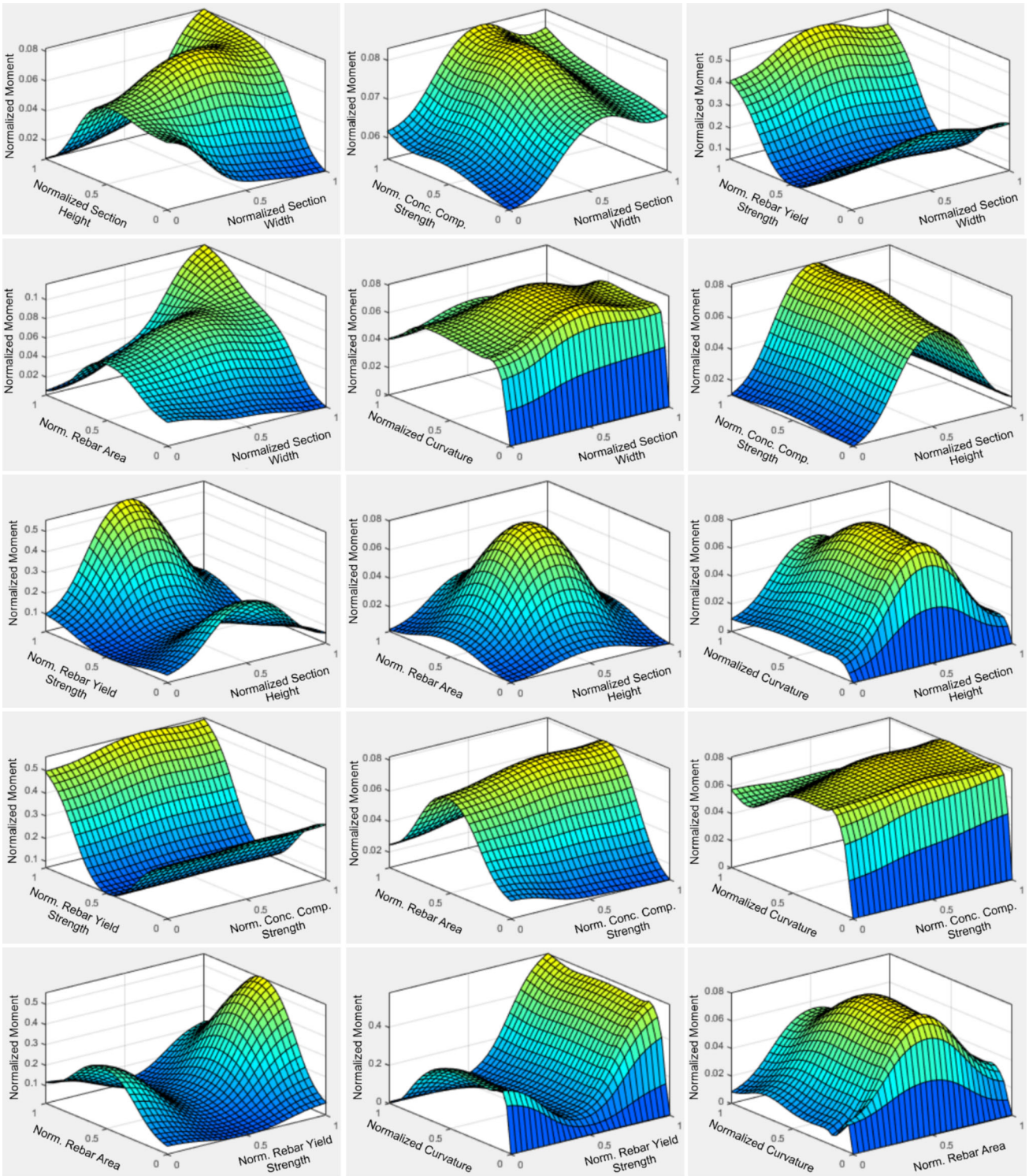


Fig. 8 Rule surfaces

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Declarations

Competing interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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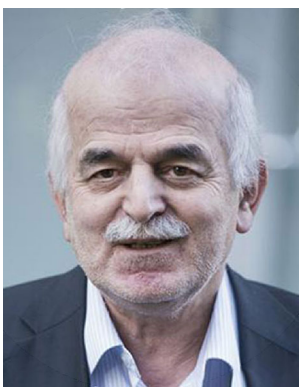
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